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INTRODUCING MATHEMATICS IN PRESCHOOL AND YEAR 1

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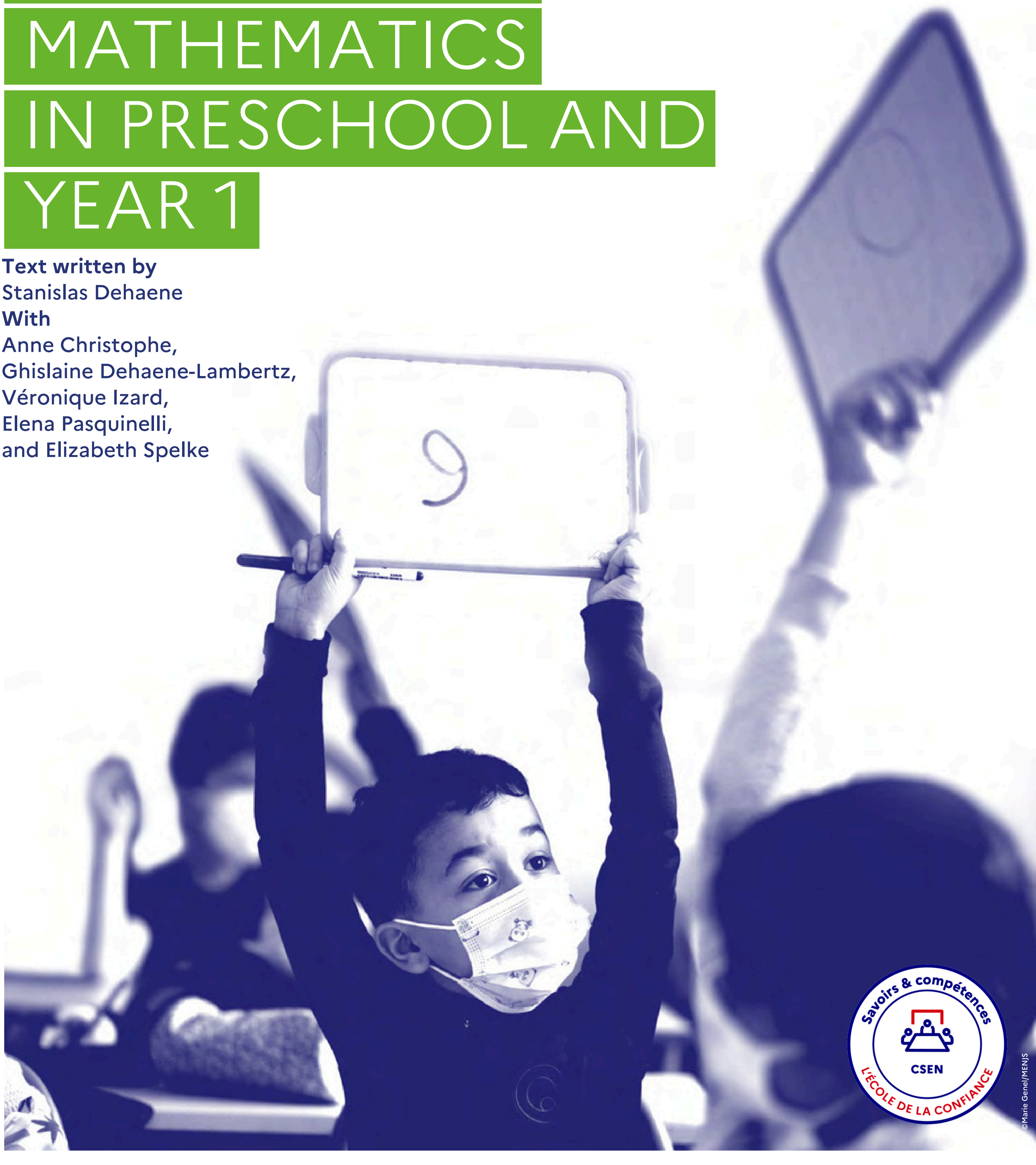


Introducing mathematics
in Preschool and Year 1

Stanislas Dehaene with Anne Christophe, Véronique Izard, Ghislaine Dehaene-Lambertz, Elena Pasquinelli, Elizabeth Spelke and the members of the Conseil scientifique de l'éducation nationale February 2021

Note. This document seeks to identify, among the findings of cognitive science research, those that we consider most relevant to the teaching of mathematics in preschool and elementary school. The current state of research in this field rarely provides ready-made solutions that can be directly implemented in the classroom. It is therefore up to teachers to draw on these findings and adapt them to their own practice in the ways they judge to be most appropriate. These pedagogical approaches could, in turn, be subjected to rigorous evaluation to assess their impact on pupils' learning.

What is mathematical thinking?

Mathematics may be defined as ***the science of patterns***.[1] Wherever precise rules exist and one reasons about their logical consequences, mathematics is present. Human beings take pleasure in discovering these rules and their sometimes unexpected consequences, and in explaining them to others.

Mathematics is also ***the art of proof.*** Explaining and demonstrating ideas through logical arguments lies at the heart of mathematics. Learners gradually acquire the ability to construct logical reasoning, which eventually becomes a mathematical demonstration—that is, a complete and convincing argument, free of gaps or ambiguity.

These two dimensions of mathematics—patterns and proof—are only beginning to emerge in preschool; many years will be required for them to develop fully. Nevertheless, preschool provides an opportunity to awaken children's awareness of both dimensions:

- Through **games and spatial construction activities**, children begin to develop an intuition for numerical and geometric patterns.
- Through **language and interactions with others**, children gradually acquire the vocabulary needed to categorize the world and to argue with increasing precision.

Looking beyond numbers

These definitions of mathematics show that the subject extends **far beyond knowledge of numbers alone.** Although numbers are a fundamental pillar of mathematics, they are not its only one. Introducing young children to the pleasure of mathematics in preschool first and foremost means **engaging them with shapes, measurement, space, puzzles, logic, sets, etc**.

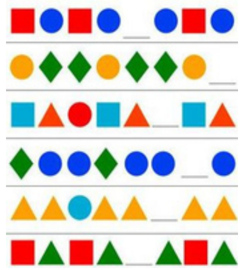
Introducing mathematics through play

In preschool, every area of mathematics can be introduced through play and the construction of tangible objects. By engaging with a wide variety of concrete situations, children discover the usefulness of the mental tools—numbers, measurement, vocabulary, symbols, and rules—that humanity has developed to solve problems. Gradually, year after year, their understanding of these same concepts becomes increasingly abstract.

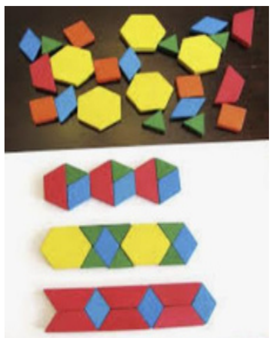
At the preschool level, all learning can be introduced through play, provided that play is designed with clear educational objectives. This does not mean allowing children to play aimlessly at all times. Research has shown that a purely discovery-based approach, without explicit guidance and structured activities, is ineffective.[2] What kinds of games, then, are appropriate in preschool?

Research identifies **spatial thinking as one of the foundations of mathematics** and shows that it can be fostered from the preschool years through a wide range of games, tools, and construction activities.[3-6]

- **Construction games** (e.g., blocks or LEGO® bricks) improve later mathematical achievement. They are especially beneficial when children are given a specific goal—for example, building a tower of ten cubes arranged from largest to smallest (the well-known Montessori Pink Tower)—or, even better, reproducing a model from a picture. Such activities require children to move from two-dimensional representations to three-dimensional objects, engage in mental rotation, compare objects at different scales, understand proportional relationships, and more. Of course, for very young children, the models can be extremely simple (for example, copying a tower with a red cube placed on top of a blue brick). Construction play also provides excellent opportunities to **discuss** space and to develop vocabulary related to spatial relationships, geometric shapes, and logical concepts (e.g., above, below, behind, cube, square, if...then, alternate, repeat).
- **Patterns and sequences** to be reproduced or completed (using stickers, LEGO® bricks, beads, …) encourage children to identify regularities in number, shape, size, and alignment. It is also valuable to ask children to draw or complete a mirror image, reproducing a model by reflection across a line. Such tasks require comparing, measuring, counting, identifying corresponding shapes…



- **Puzzles** help develop shapes' spatial visualization and mental rotation skills, in addition to fine motor control and visual attention. Research shows that experience with puzzles enhances later spatial reasoning and mathematical abilities.[3]
- **Activities involving geometric shapes**—such as circles, triangles, squares, and diamonds (rhombuses)—take many forms, including sorting activities, tangrams, and reproducing mandalas. These activities develop children's understanding of space, orientation, measurement, and proportion, while enriching their geometric vocabulary and manual precision. Particular care should be taken to present a wide variety of shapes : research shows that children may otherwise only figures the prototype of a triangle (typically an upright equilateral triangle).[7] They should therefore encounter many different kinds of triangles, for example by sorting them into categories.



- **Card games,** such as War, and **board games involving buying and selling** such as *Pay Day* help children estimate and compare numbers, recognize numeral symbols, and develop intuitions about numerical magnitudes as well as addition and subtraction. Preschool children's ability to compare numbers is a strong predictor of later mathematical achievement in first grade.[8-9]

- **Counting games** develop number sense as well as numerical and spatial working memory, both of which are excellent predictors of later mathematical success.[4] For example, a teacher may hide several objects in a box, remove some of them, and ask children to determine how many remain. This simple "mystery" can increase children's interest in numbers and counting, themselves strong predictors of subsequent mathematical development.[10]
- For more advanced preschoolers, **linear board games** such as *Snakes and Ladders* or similar games, in which a playing piece advances a number of spaces determined by a die roll, teach the concept of the mental number line, exact numerical magnitude, counting, addition, and subtraction. Children who regularly play such games make faster progress in mathematics, particularly in understanding the meaning of numbers.[11-14] These benefits are even greater when children are encouraged to say aloud the numbers printed on the spaces as they move their playing pieces.[12]

In preschool, there is no meaningful distinction between playing and working. Everything is play, and everything is serious learning. For this reason, activities involving space and measurement—using tools such as rulers, measuring tapes, and scissors (and later compasses, set squares, and protractors)—are also part of introducing children to mathematics. Throughout the preschool years, children can engage in activities such as:

- **cutting out and constructing** cardboard objects, which thereby leads children to manipulating plans and geometric shapes, measuring and comparing lengths;
- **measuring** not only lengths but also diameters, masses, volumes, and temperatures (a thermometer provides an excellent introduction to the number line and to negative numbers);
- **sorting** objects by size and collections by quantity;
- **drawing** shapes and objects with increasing precision;
- **constructing** objects using modelling clay, DUPLO® bricks, and similar materials.

The benefits of of practicing mathematical games are numerous:

- A play-based learning approach creates a gentler school environment, in which children enjoy having fun together.
- Play is an excellent **remedy for maths anxiety**, fear or even a dislike of maths that develops in many pupils. Indeed, through play, we discover that mathematical concepts are not daunting, but playful, a source of pleasure and beauty
- Playing with others fosters **language development and social-behavioral skills**, including self-regulation, listening to others, and constructing appropriate arguments.
- Mathematical games need not be competitive; they can also be cooperative.[15] When children work together toward a common goal in small groups, they challenge one another's thinking, share strategies... Research shows that peer instruction benefits all participants, including both the child who teaches and, evidently, the child who learns.[16]
- Games, and particularly solo games, also foster **self-confidence**, particularly when children repeatedly return to the same activity and experience their own progress toward mastery. Numerous studies show that enjoying mathematics—or believing oneself to be "good at math"—is strongly associated with later mathematical success.[17,18]

Approaching the same mathematical concept from multiple perspectives

Models of learning show that learning is faster when tasks and learning contexts are varied.[19] It is therefore beneficial for children to encounter the same mathematical concept through a range of different activities. For example, the concept of number can be introduced in several ways:

- as the cardinality of a set of objects;
- as the number of sides of a geometric shape (triangle, square, pentagon, etc.);
- as the ordinal position of an item in a sequence;
- as a word in the counting sequence;
- as a measure of space or length;
- as an amount of money in the Pay Day game.

Developing a deep understanding of numbers requires children to move flexibly between these different representations of number.[20] They learn to connect quantities with numerals, numerals with number words, ordinality with cardinality, and so forth. By varying the contexts, we help children gradually abstract what these different games, tools, situations, and problems have in common.

It is important to note that diversifying learning contexts does not mean mixing them together within a single lesson ! During an individual lesson, it is often helpful to focus children's attention on a minimal contrast—a pair of situations that differ in only one respect. Such contrasts help children notice the relevant distinction (for example, "*Look, this carrot is big, whereas this one is small*," or "*Here there are two triangles, while here there is one more—three triangles*."). However, even if children understand the difference, a single example is unlikely to be sufficient for them to retain the underlying general concept. Multiplying examples day after day helps them construct the abstract principle (for example, "Each time I add one object, the number increases by one."). This idea can be summarized in a simple principle: **multiply the number of minimal pairs**.

Adopting a spiral approach

Because abstract mathematical concepts emerge very slowly from a multitude of concrete situations, it is useful to **adopt a spiral approach**. Children are introduced to advanced concepts at an early age, even though they may not fully understand them at first. These concepts are then revisited year after year, in increasingly varied and new contexts, with understanding deepening each time.

Fractions provide a good example. Around the world, learners find it difficult to understand how the ratio of two numbers, such as ½ or ¾, represents a new number, what such fractions mean, and how they can be manipulated.[21] In the French curriculum, fractions are introduced formally only in the later years of primary school (typically in Grade 5), and evidence shows that they remain poorly understood by many students when they enter lower secondary school. Yet, in everyday language, children typically learn words such as half and one-half much earlier, around the age of five ![22]

From the preschool years onward, children can encounter fractions in many concrete situations that they understand intuitively: sharing a cake into two equal halves, dividing a collection into two equal groups, asking for half a glass of water, or buying something at half price. Terms such as one-third and one-quarter can naturally be introduced somewhat later. These early intuitions, developed during preschool, provide a foundation that greatly facilitates later learning when the same mathematical concepts are presented in a more symbolic and rigorous form.

Teaching the rich language of mathematics

The example of simple fractions illustrates why it is a mistake to view the teaching of language and mathematics. Across all domains of learning, language serves as a catalyst for conceptual development. It directs children's attention to important properties and encourages them to make distinctions that might not otherwise be obvious.

For this reason, children should frequently hear language related to numbers, shapes, logic... **Mathematical thinking relies on a rich and specific vocabulary :**

- add, subtract, divide, alternate, repeat;
- near, far;
- in front of, behind, above, below, on, under, inside, outside;
- square, circle, rectangle, triangle;
- first, second, third;
- half, third, quarter;
- few, many, more, less, most, too many;
- before, after, during;
- true, false, certain, probable, perhaps;
- regular, frequent, rare, uniform, different, identical, symmetrical.

Research shows that the better children understand this mathematical vocabulary, the stronger their skills in elementary arithmetic.[23]

To enrich children's mathematical vocabulary, teachers can read **stories with mathematical content**—for example, Goldilocks and the Three Bears. The French researcher Michel Fayol has also developed a series of stories that gradually increase in numerical complexity, for instance by describing a growing family that needs more and more pairs of shoes for example.

Teachers should also "think aloud"—that is, make their reasoning explicit by verbalizing their thought processes as precisely as possible while using rich, accurate, and carefully chosen vocabulary.

Introducing mental tools, linguistic and non-linguistic

Words are only one example of the **symbolic** tools that captures mathematical thinking and enables it to develop. Other examples of mathematical symbols also include :

- the Hindu-Arabic numerals;
- operation symbols such as +, −, ×, together with parentheses, which form the basis of algebra;
- the dot patterns representing the numbers 1 to 6 on dice;
- the Asian abacus, in which each number is represented by a certain number of beads arranged in fixed patterns ;
- the number strip, in which each successive whole number (1, 2, 3, ...) occupies one additional space, and the number line, in which every number is represented by a precise location;
- graphical representations. In the bar model approach promoted by Monica Neagoy,[24] building on teaching methods widely used in Singapore, an unknown quantity is represented by a bar of a given length. By partitioning or scaling these bars, children can solve one-unknown problems well before they receive formal instruction in algebra.

It is, of course, unrealistic to expect preschool children to master all of these concepts, useful metaphors, and "mental tools." What matters is that the diversity of situations and representations they encounter gradually equips them with a repertoire of tools for their entire schooling. In keeping with the spiral approach, we can safely start at an early stage to build familiarity with those tools, without expecting every detail to be fully understood immediately. Research shows, for example, that preschool children already begin to grasp the structure of numbers beyond twenty[25] and can approximate the results of two-digit additions.[26]

Nor should there be concern that enriching the preschool curriculum—particularly in the final preschool year—with mathematical content that anticipates elementary school topics would come at the expense of children's social and emotional development. **On the contrary,** research conducted with children in the final year of preschool shows that exposure to more advanced content is associated with stronger interpersonal skills, improved attention, and more positive attitudes toward learning, especially among children who initially entered preschool with fewer advantages in these areas.[27] These findings are consistent with research on metacognition, which demonstrates that children's sense of self-efficacy, motivation to learn, and academic self-concept all increase as they gain mastery in a domain.[28]

Regularly assess each child's progress

In preschool, children's abilities, prior knowledge, and learning needs vary considerably depending on their age and previous experiences. Learning activities should therefore be situated within each child's zone of proximal development—that is, just beyond what the child can already accomplish. This is where they find enjoyment: tasks that are too easy quickly become uninteresting, whereas those that are too difficult are discouraging. It is therefore essential to be able to assess precisely where each child stands in their progression through mathematical concepts, without simply relying on one’s own impression. Research has shown that when teachers regularly monitor children's learning, progress accelerates in both language and arithmetic.[29] The following examples summarize the assessment tasks proposed by these researchers to evaluate children's understanding of numbers.

Cardinality	Show with your fingers how many objects appear on a page Produce a specified number of objects.
Counting	Point to objects one at a time while saying the corresponding number words; Recite the counting sequence in the correct order.
Knowledge of numerals	Name each of the ten Hindu-Arabic numerals (0–9).
Comparison of numbers and sets	Decide which of two hands is showing more fingers; Arrange cards depicting one to five objects (or fingers) in ascending order.
Arithmetic operations	Predict the result after objects are added to or removed from a cup; Understand simple stories involving the addition or subtraction of objects.

Discovering numbers and arithmetic

The domain of number is the area of mathematics that has been studied most extensively by cognitive science. As a result, researchers now have a fairly detailed understanding of the developmental stages through which children acquire numerical competence. This section summarizes the principal findings.

What children already know. Children entering preschool are far from being devoid of numerical abilities. Research shows that, from birth, all children possess a basic understanding of objects and an innate number sense. They can recognize collections of one, two, or three objects without counting—a capacity known as subitizing—and they can also discriminate between larger quantities when the difference is sufficiently large: even infants can distinguish between four and twelve objects ! Babies also possess rudimentary intuitions about numerical order, enabling them to appreciate that twelve is greater than four. Finally, they have an approximate understanding of addition and subtraction. For example, experiments have shown that infants who see five objects hidden behind a screen, followed by five more, expect approximately ten objects to be revealed when the screen is removed.

The mental construction of mathematics builds upon these early abilities in subitizing (for the numbers one, two, and three) and approximate estimation. Together, they provide preschool children with an informal intuition for numbers that gradually develops into mature number sense. Thus, even if a preschool child does not yet understand the meaning of the number word twenty-four, they may nevertheless appreciate that it refers to a large quantity—although a smaller one than forty-two.[26] **The meaning of mathematical symbols is therefore grounded in children's early intuitions about quantity.**

What children need to learn, and how. Approximate numerical intuition alone is not sufficient. During the preschool years, children begin to acquire **the precision that characterizes exact number**. The introduction of numerical **symbols**—both number words and written numerals—and the ability to connect these symbols fluently with the quantities they represent are among the strongest predictors of later mathematical achievement.
This learning process unfolds along several dimensions.

1.Separating number from other dimensions

One of the first conceptual advances made during the preschool years is the realization that number is an ***abstract property***. Collections may contain the same number of items even if they differ in every other respect. Children should therefore learn to count collections of objects, sounds, people… in all kinds of situtions—for example, the number of cars passing by, or the number of people entering and leaving a room).[30] Gradually, they learn ***to focus specifically on numerosity*** rather than being distracted by irrelevant features such as the size of the objects.[31]

2.Understanding set equality and the effects of transformations

Understanding arithmetic means understanding how a set of items changes when objects are added or removed. Young children quickly grasp one-to-one correspondence: if two sets can be perfectly matched item by item, then they contain the same number of elements. They take longer, however, to understand that some transformations change the number whereas others do not—for example, replacing one object with another leaves the number unchanged.[32] Playing with paired collections—for instance, dolls and their hats—help children develop not only an understanding of numerical equality but also the intuition that adding or removing an item from one of two equal collections destroys this equality, whereas opposite operations (such as +1 followed by −1) restore it. Replacement situations require an even higher level of numerical abstraction to solve the problem. Imagine that two hats become dirty, leaving two dolls without hats, and children are charged with determining exactly how many replacement hats are needed—no more and no fewer. Similar activities involving animals and socks can also introduce intuitive ideas about doubling, quadrupling, or dividing by two or four.

3. Learning to count accurately

Counting a collection accurately is a genuine challenge for children, and should be firmly established during the preschool years. Successful counting relies on several abstract principles.

- ***Fixed-order principle.*** Number words must always be recited in the same order: one, two, three, four..., without omissions. This « number rhyme » should be mastered as early as possible, typically right at the start of preschool (bearing in mind, in particular, children who speak a different language at home).
- ***One-to-one correspondence principle***. Every object must correspond to one and only one number word. Children must neither count an object twice nor omit one. Counting establishes a perfect correspondence between each object and each successive number word. In preschool, this principle can itself become a game: Can you count from right to left? From top to bottom? Can you skip an object and come back to it later?
- ***Cardinality principle***. Regardless of how objects are counted, the final number word has a special status: it is the cardinal of the set, representing the total quantity that has been counted. It answers the question How many? Young children do not always appreciate the unique significance of this final number. They must learn that counting objects one by one is the reliable way to to answer the question 'how many objects are there?'. Counting, in order to arrive at the precise cardinal number of a set, must become second nature to preschool children.

4. Learning number symbols and their meaning

The acquisition of numerical symbols—including cardinal number words (one, two, three), ordinal words (first, second), written numerals (1, 2, 3...), and multi-digit numbers (forty-two, 53)—is a fundamental component of mathematical development that begins in preschool and continues throughout elementary school. The rapid, automatic, and effortless conversion between symbols and the quantities they represent during the early school years is a strong predictor of later mathematical achievement.[33]

What exactly should children learn? First, they must learn the forms of the symbols themselves. Being able to recognize and write the ten Hindu-Arabic numerals, as well as reciting the counting sequence fluently, are expected outcomes by the end of preschool. However, ***this simple recitation alone is far from sufficient***. Research shows that children may appear highly competent because they can recite number words rapidly, almost like a poem, while having little understanding of what those words actually mean.[34] Preschool must therefore ***give meaning to numbers.*** Between approximately two-and-a-half and four years of age, children gradually begin to connect number words with the corresponding quantities ("three" means exactly three objects). Two simple exercises can be used to assess this ability to understand:

- "***Give me a number."*** A collection of objects (for example, dolls) is placed before the child, who is asked, "*Give me two dolls*," "*Give me three dolls*," and so forth. A child who understands the meaning of three consistently gives exactly three dolls. A child who does not yet understand typically hands over a random handful.

- "***How many do you see?***" Children are shown a precise set of items, either physically or in a picture, and asked, "*How many dolls do you see?*" or "*How many LEGO® bricks are there?"* A child who has grasped the meaning of numbers is able to state, precisely, the cardinal number of the set (for numbers above 3, they will need to count). A child who has not grasped this will give a random word.

Progress is very slow : during the first preschool year, many children succeed only with the smallest numbers, such as one and two. After several months, they master three, then four. Eventually, an important conceptual breakthrough occurs: children realize that every number word in the counting sequence corresponds to one precise quantity. Every number word has a unique meaning. This learning is foundational : until children achieve it, many later mathematical concepts remain inaccessible.[35] Preschool teachers can facilitate this transition by consistently using numerical language with precision and by encouraging the same precision in children's use of all mathematical vocabulary.

5. Learning to compare numbers

One of the clearest indicators of numerical understanding is the ability to determine which of two numbers is larger or smaller. Comparison is among the most fundamental skills in arithmetic and one of the strongest predictors of later mathematical achievement.[8] Children with developmental dyscalculia likewise often experience particular difficulty with numerical comparison.

During the preschool years, comparisons should primarily involve concrete quantities—collections, lengths, etc. —while simultaneously introducing the corresponding number words or numerals. At this age, isolated symbols remain difficult to interpret without contextual support. Indeed, by around four years of age, children solve simple additions and subtractions much more successfully when problems are presented concretely with objects than when presented verbally or symbolically using written numerals. Only later do they become able to compare symbolic numbers directly (for example, two numbers written in Arabic numerals).[36] This ability depends on automatically linking each numeral to its corresponding numerical magnitude, a process that gradually becomes automatic during the first years of elementary school.[37]

6. Learning to compose and decompose numbers

Through repeated manipulation of sets of items, children gradually develop a fluent understanding of relationships among numbers. In preschool, this concerns only the smallest numbers: 1, 2, 3 and 4. « Three lies *between* two and four. » , « Four is two *plus* two, or three *plus* one. ». Over time, this network of knowledge expands: « *One hand has five fingers.* », « *Two hands have ten fingers—that is, two groups of five*. ». Children who possess this rich and fluent understanding can use it to solve problems more efficiently—for example, by counting in groups of two. Research shows that the ability to recognize and count groups—for instance, immediately seeing that three groups of two make six—is an important predictor of later arithmetic performance.[38]

Many different activities can foster this understanding. It is fun, for example, to ask the child to predict what will be inside an opaque box after objects are added one by one or two by two, or listen to a story in which three bears each need two socks. ***Varying the situations and transformations*** helps children to understand the abstract nature of number sense.

7. Building a mental model of numbers: The number line

Numbers are used not only for counting but also for ***measuring space***. Research shows that, as adults, we cannot think of a number without visualising a position in space, on a sort of 'mental number line'. During the preschool years, children discover that numbers can be arranged on a line, from left to right. Each whole number occupies its own location, together forming a number strip. This representation greatly facilitates arithmetic comprehension, as addition corresponds to movement to the right and subtraction to movement to the left. Later, children can progress to the more sophisticated concept of a graduated number line, on which every number occupies a precise position. This representation supports understanding that other numbers exist between whole numbers —for example, decimals and fractions.

The conceptual transformation that occurs during the preschool years should not be underestimated. Young children—and even adults with little formal education—do not initially appreciate that numbers are *evenly spaced* along the number line, with the same distance separating every pair of consecutive integers. Instead, they often perceive small numbers as distinct while treating larger numbers as increasingly similar. Acquiring the concept of the number line—a genuine mental model of numerical space, a kind of internal "slide rule"—marks a major milestone in children's mathematical thinking.[39]

National assessments at the beginning of first grade indicate that many French children have only a limited understanding of the number line. How can this be strengthened? Firstly, from preschool onwards, a number line can be displayed in the classroom showing, in very large numbers from left to right, the numbers from 1 to 10, or even 100: each of these numbers occupies a different square, rather like in the game of the goose. Teachers can then refer to this representation continually when discussing quantities, measurements, ages, and so on in relation to one another.

Furthermore, many of the games described throughout this report also contribute to children's understanding of the number line. Linear board games such as *Snakes and Ladders*, in which children move a playing piece a number of spaces determined by a die roll, have been shown to promote later arithmetic learning.[11,40] Other games involving measuring space with a ruler (to cut out a square of a given size, for example), sorting items into compartments, or using coordinates to guess where a hidden object is (such as 'Battleship'), are likely to have a similar effect. The variety of situations that require matching a number to a length or a position in space highlights the fundamental nature of this concept in mathematics.

KEY TAKEAWAYS AND PRACTICAL RECOMMENDATIONS

- In preschool, all areas of mathematics can be introduced through **play** and the **construction** of tangible objects.
- Introducing children to the **pleasure of mathematics** means engaging them not only with numbers, but also with **construction** activities, geometric **shapes**, **measurement**, **space**, **cards**, **puzzles**, **logic**, and **sets**.
- To facilitate the transition to abstract thinking, teachers should revisit the same mathematical concept regularly (for example, the contrast between two and three, or the concept of an even number), while approaching it from new perspectives and **varying the games and learning contexts**.
- Even relatively advanced concepts—such as numbers beyond twenty or simple fractions (half, one-half)—can be introduced **early** and revisited each year in progressively greater depth, following a **spiral approach**.
- Mathematical thinking relies on a rich and specific **vocabulary**. Teachers should use this vocabulary consistently across all mathematical activities and should **think aloud, making their reasoning explicit for children.**
- **Regular assessment**—even through very simple tasks (Can this child produce three objects? Does the child understand before and after, above and below?)—enables instruction to be more closely tailored to individual learning needs.
- **Instructional progression** should be informed by what is known about children's cognitive development. In the domain of number, this trajectory includes learning to:
  - **focus attention** on the number, and distinguishing it from other dimensions;
  - understand the **equality of sets** and the effects of transformations on quantity;
  - **count** accurately;
  - understand both **spoken and written numerical symbols** and their meanings;
  - **compare** and order numbers;
  - **compose** and **decompose** numbers;
  - develop a mental representation of number through the mental **number line**.

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