MINISTÈRE
DE L'ÉDUCATION
NATIONALE
Liberté
Égalité
Fraternité

Conseil scientifique
de l'éducation nationale

Note from the CSEN —
— June 2023, n°10

Patterns: a gateway to mathematics in preschool and primary education

Written by Lorenzo Ciccione and Stanislas Dehaene

Abstract

How can we stimulate an interest in mathematics from an early age? In early childhood education, children are often asked to create or complete patterns, such as threading a yellow bead, then a red one, then a yellow, then a red, and so on, on a necklace. These activities are sometimes seen as exercises to develop fine motor skills, writing, or artistic expression. However, we demonstrate that they are primarily a powerful stimulus for the development of mathematical skills, particularly in geometry and logic.

Indeed, the study of patterns leads children to form numerical and geometric abstractions that they can transfer from one domain to another. Recognizing the same pattern in a sequence of musical notes and in a row of beads draws the child's attention to the abstract properties of numbers, symmetries, rules, and written notations. This is why we propose making pattern-based activities more systematic in early childhood and the early years of primary education, and we present a hierarchy of activities that can be used in the classroom.

What is a pattern?

Every early childhood teacher knows that children love arranging colored blocks, alternating musical sounds or vocal patterns in a regular manner, or repeating graphic symbols one after another as soon as they learn to hold a pencil. When asked about these activities, teachers report that they are essential for developing fine motor skills, writing, and aesthetic and musical sense. But what do all these activities have in common? They use what are generally called "patterns" (or "motifs" in French), meaning sequences or configurations of objects (visual, auditory, or even tactile) organized spatially or temporally according to defined rules. In other words, these are sequences in which we recognize a certain order and predictability, to the point that we could extend them. Here are a few examples (Figure 1): blue cubes alternating with yellow cubes along a line; the high-pitched sound of a whistle alternating with the low-pitched sound of a drum, while gradually increasing the number of repetitions of identical sounds; or geometric shapes repeating on wallpaper.

Whatever the medium, we are constantly surrounded by a variety of patterns. It is therefore not surprising that understanding patterns is included in the national curricula of France and many other countries. Under the term "graphism," many textbooks propose creating repeated geometric patterns, often as a preparatory activity for writing (Figure 2), and many early childhood and primary school teachers use them during their lessons. Scientific research from recent decades supports teachers: playful activities with patterns seem to enhance abstraction skills far beyond simply improving writing or aesthetic sense, as we will see in the following sections. But what do patterns have to do with abstraction?

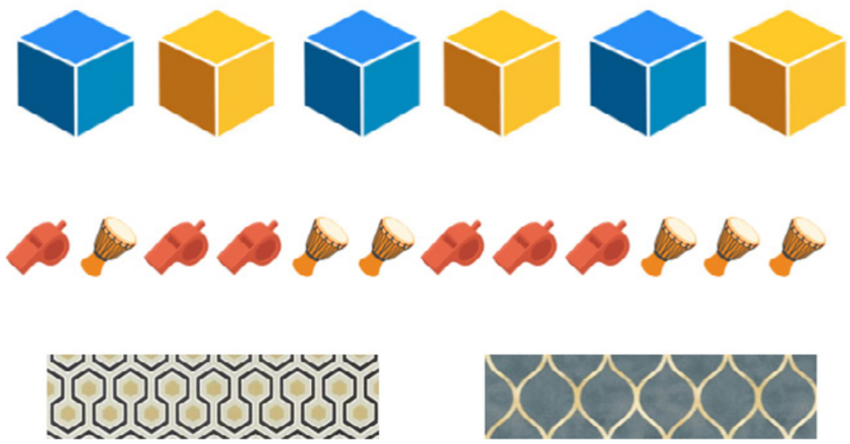

Figure 1. Examples of patterns: a series of alternating colored cubes; alternation with increasing repetition of high-pitched and low-pitched sounds; wallpaper patterns.

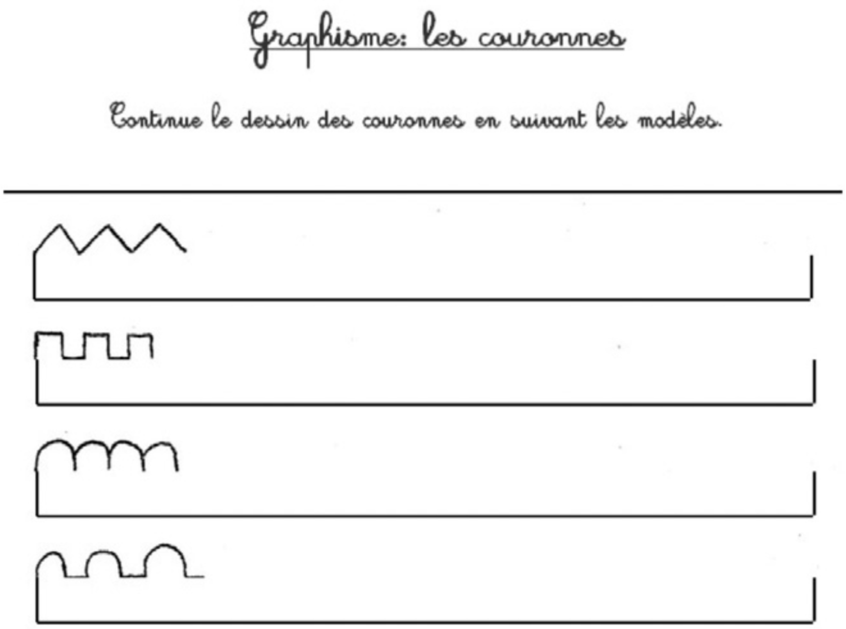

Figure 2. Example of an activity to improve "graphism" (fine motor skills for writing)

Different domains, same abstract structures

Children (and adults) encounter patterns in very different domains: drawing, writing, music, and mathematics, to name just a few. However, whether they are visual, auditory, or mathematical sequences, the abstract structure of the pattern can be the same. In each pattern, we can recognize a minimal unit (points, blocks, numbers, sounds) that is repeated in space and/or time according to one or more precise rules. To clarify the concept, let's take two examples: one visual, the other auditory. As we can see in Figure 3, a red square is followed by a blue square (which in this case form a "minimal unit"), followed in turn by two red and two blue squares, and finally three red and three blue squares; in this pattern, the rule is clear: we alternate two different colors, but with each alternation, we add one square of each color, so that their number increases: 1,1,2,2,3,3.... Now let's take the auditory example from Figure 3; the alternation of the two notes is determined by the same rule used for the sequence of colored blocks: do, fa, do, do, fa, fa, do, do, do, fa, fa, fa, etc. In this example, the minimal units have a different appearance (colored squares or musical notes), but the rule is the same: in other words, these patterns are two distinct

Glossaire

Pattern: An ordered arrangement of elements that repeats according to a certain rule. These elements can be objects, two- or three-dimensional geometric shapes, sounds, numbers… Two patterns can have a different physical manifestation (for example, a sequence of sounds or a series of colored blocks) but be organized according to the same abstract rule (for example, the alternation of two elements one after the other: ABABAB).

Rule: An abstract formula that determines how the elements of a pattern are organized. The rule is abstract, independent of the physical pattern by which it is illustrated: the same rule can be represented by a pattern composed of sounds, objects, numbers, etc. Recognizing the underlying rule of a given pattern allows us to predict its evolution. There are simple rules (such as the repetition of the same element—AAAA—or the alternation between two elements—ABAB) and more complex rules, which provide, for example, an increasing progression in the number of appearances of each element (e.g., ABAABBAAABBB), pairs, triplets, symmetry… or even a nesting of these concepts.

Mental Program: A mental representation that a human makes of the rule to which a pattern obeys. Such a program allows compressing a pattern so that it takes up less space in memory (ABABABAB… = "an alternation"). It structures and guides thoughts and actions related to the presented pattern (for example: copying it, transposing it, or completing it). A mental program may not respect the true rule underlying the pattern, especially when it is complex and difficult to compress.

physical manifestations of the same rule, which could be noted as the abstract formula "ABAABBAAABBB."

It is therefore clear that **the notion of a pattern is profoundly abstract**, as it does not depend on the concrete unit to which it is applied. Furthermore, the recombination of a limited number of rules can give rise to superficially very different pattern manifestations. Finally, it is important to emphasize that the recognition, memorisation or (re)production of a pattern are obviousky determined by the level of concreteness or symbolic abstraction of its minimal unit – manipulating colored blocks is not the same as manipulating a sequence of numbers – as well as by the quantity and typology of rules applied to it – the notion of « repetition » is certainly more inutuitive than that of « increasing number ».

Mathematics: the science of patterns

Modern mathematics is often defined as the science of patterns or regularities[1]: one could almost say that the goal of mathematics is to find, formalize, and reuse patterns in the world around us. While algebraic formalization (i.e., using letters instead of numbers) is difficult for children just beginning to study mathematics[2], patterns are often used to introduce algebra in an intuitive way[3].

Scientific research has proven that humans perceive mathematical patterns very early on. When shown a geometric shape, asked to copy a drawing, or made to hear a series of sounds, both adults and children quickly grasp the abstract logic (see the box on "Cognitive psychology and neuroscience of mental programs"). The simpler the pattern, the better their memory (for example, an ABAB... alternation is easier to remember than an AABBAABB... sequence)[4]. Enumerating a collection of objects is easier for them when they can mentally organize it into repeating subgroups[5], particularly if these subgroups are arranged according to the same visual pattern[6]. They are capable of recognizing regular functions, whether linear (a straight line) or not (a zigzag), when represented in graphical form[7–11]. They can even combine geometric shapes to form, for example, "a square made of small circles"[12], a highly abstract concept that surpasses what other animals can conceive[13].

This understanding is intuitive, "proto-mathematical": it requires no formal knowledge of algebra or geometry. On the contrary, the most recent scientific research suggests that, from early childhood, engaging in playful and intuitive activities focused on recognizing and using patterns can facilitate the development of not only visuo-spatial skills and reading/writing abilities but also mathematical learning[14]. In the following paragraphs, we first present the developmental trajectory of understanding and manipulating

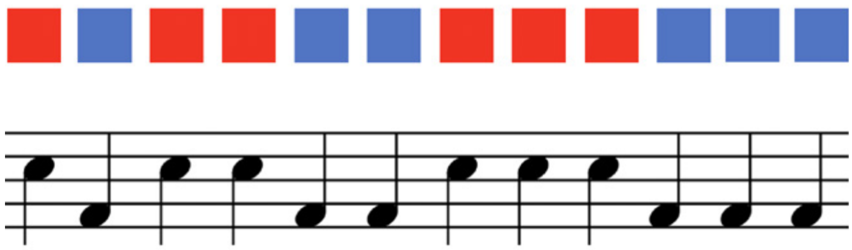

Figure 3. Pattern following the same rule but represented in two different modalities: visual and auditory.

proto-mathematical patterns, then we focus on studies suggesting a link between pattern comprehension and mathematical skills.

Understanding patterns across child development

If the visual and auditory recognition of simple sequences is possible from birth[15], it is only from the age of 3 (i.e., when starting preelementary school) that the child shows the ability to recognize and produce patterns. At the age of 3, children are able to recreate simple combinations of 2 to 3 blocks by copying a given model[16] (Figure 4, left). At 4 years old, they can reproduce, extend and abstract different patterns such as ABAB, AABBAABB, and ABBABB[17] (Figure 4, center), but not all of them are yet able to extract and indicate the minimal unit that is repeated. Furthermore, at 4 and a half years old, when presented with simple patterns such as those in Figure 4, they are able to copy them using elements different from those to which they were exposed (i.e., different shapes and/or colors; Figure 4, right). This transposition of a rule from one medium to another is facilitated when the patterns are described using abstract labels (for example, with the letter series ABAB)[18].

They are also better able to generalize from three-item patterns, such as ABA, when their attention is drawn to the lateral position of the two marginal elements relative to the central element (Figure 5)[19]. These two studies clearly suggest that working on the abstract dimension of patterns is not only possible but also useful from the first years of preelementary school. However, at the age of 4, children still seem unable to extend patterns with increasing numbers, such as those described earlier in Figure 1 (middle line)[20], which suggests that their set of mental programs is limited to simple operations of repetition and alternation. At the age of 4, they also struggle to recognize the same ABA pattern if it is presented with elements that differ in more than one dimension[19] (Figure 5, right): in other words, their abstraction skills do not seem to withstand too significant variations in the appearance of the stimuli. This task is correctly performed at the age of 6.

Indeed, it is from the age of 5/6 that the ability to manipulate patterns explodes: at this age, children are now able to extend a sequence of objects organized according to different patterns, regardless of the physical characteristic that varies (shape, color, size...[21]).

Figure 4. Progression of children's abilities. At age 3, children can copy a pattern composed of a few elements. At age 4, they can extend different patterns (with different minimal units subject to repetition, indicated in red): ABAB, AABBAABB, and ABBABB. Around age 4 and a half (and older), they can also transpose a pattern, meaning they can reproduce it using different shapes.

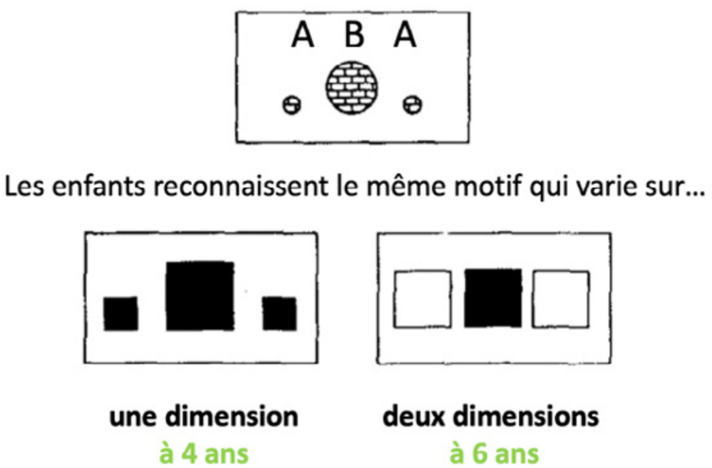

Figure 5. The top pattern has the same abstract structure as the bottom patterns (ABA). In the first, the size ratios are respected; in the second, the size ratios are not respected, and B has a different color from A.

Link between understanding patterns and spatial and mathematical skills

Several studies suggest that there is a close link between the ability to understand patterns and spatial and logical-mathematical skills. We mainly distinguish two types of studies: correlational studies (which find correlations between the ability to understand patterns and other cognitive abilities) and interventional studies (which seek to measure whether training in the former can lead to an improvement in the latter).

In the specific case of colored bricks like Lego, which are among the most commonly used tools in education to stimulate pattern comprehension, it has been observed that the time children spend playing with them is correlated with the spatial abilities of 5.5-year-olds[22]. At age 3, the ability to copy patterns presented as colored blocks is correlated with numerous numerical comprehension tasks, including enumeration, number ordering, and nonverbal arithmetic[16].

Pattern comprehension skills also appear to be correlated with proportional understanding (tasks involving the comparison and reproduction of proportions presented in nonsymbolic forms[23]). Even among older children (10 years old), the ability to manipulate numerical patterns continues to correlate with mathematical performance[24].

It is important to note that the studies presented so far cannot rule out the possibility that the link between pattern comprehension and mathematical abilities is explained by a general improvement in visuo-spatial or intellectual capacities. There is indeed a strong correlation between the ability to copy and extend patterns and visuo-spatial working memory[25] (the cognitive function that allows us to retain multiple objects and their positions in memory). However, two recent studies seem to indicate a strong and specific correlation between these two abilities, even when accounting for intelligence[26] and spatial skills[14].

Even stronger evidence of the existence of a link with mathematical skills comes from interventional studies, where students are trained and it is verified whether this learning induces an improvement in other skills—only these studies can indicate to us whether there is a causal link between two or more cognitive functions. Six months of training (15 minutes, three times a week) for 6-year-olds, aimed at stimulating the understanding and manipulation of different types of patterns (including repetitions, rotations, symmetrical transformations, reading the time, etc.), led to a significant improvement in their skills in this area[27] and also to improvements in mathematics and reading[28] compared to students who had received in-depth instruction in other school subjects. Four short lessons on patterns and the concept of the minimal repetition unit, given to 9-year-old students, also appear sufficient to facilitate discussions about fractions and proportions[29]. However, these results should be interpreted with caution, as they are not always replicated: indeed, a recent study showed that 30 minutes of weekly training for 20 weeks using patterns based on repetition and increasing number rules led to improved comprehension of these patterns, but without significant transfer to broader mathematical knowledge[30].

However, it is important to note that many authors warn readers against the risk of overestimating the results presented so far: most studies indeed show weak or moderate correlations[31]. However, much more significant effects could be observed if different, more abstract objects than those generally used in preelementary school were used. Activities involving the production of bead necklaces or sequences of colored bricks, especially if practiced excessively with the same ABABAB alternation pattern, do not do justice to children's abstraction skills[14].

Beyond visual patterns: Music

The studies presented so far have used visual patterns. But humans are perfectly capable of learning the underlying rules of complex auditory patterns, such as binary sequences of sounds[32]. Children are exposed to musical patterns from a very young age: one need only consider nursery rhymes (an example is provided in Figure 6), which are often structured according to repetition and alternation rules.

From just a few months after birth, children are naturally attracted to simple musical rhythmic patterns33, and it has also been suggested that listening to and reproducing nursery rhymes containing expressions organized according to defined patterns could foster an interest in mathematics[34,35]. The visual representation of auditory patterns also appears to improve musical skills[36], and both types of representations share, at least in part, the same brain networks[37]. This once again suggests the importance of presenting the same patterns through different sensory modalities to help children move toward abstraction.

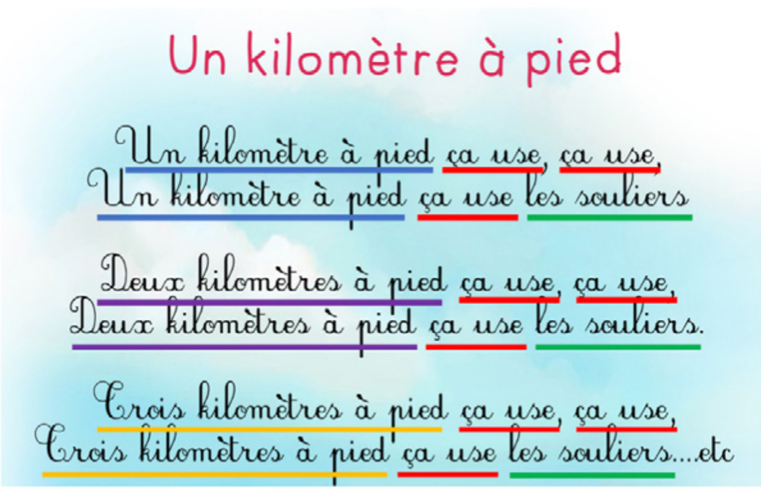

Figure 6. Example of a nursery rhyme following a pattern created by a rule of repetition, alternation, and novelty. To facilitate pattern recognition, the same elements have been underlined in the same color. The pattern is as follows: AXX AXY, BXX BXY, CXX, CXY, etc.

However, experimental results are mixed: some studies have suggested that intense musical practice improves mathematical skills[38,39] and, more specifically, seems to enhance the ability to manipulate geometric shapes and symmetrical patterns[40]. Other randomized studies, however, have shown no improvement in mathematical skills following musical training[41].

Musical practice might also have effects that extend beyond mathematics, improving reading skills[42] (which also benefit from training aimed at distinguishing recurrent letter patterns[43]). Musical practice is also correlated with grammatical comprehension in school-age children[44]. While it is reasonable to think that music and mathematics share something unique compared to other disciplines (the need to understand and manipulate patterns), the data are still very limited, and further research will be needed to explore the true nature of the link between patterns, music, and mathematics.

Recommendations for Teachers

Based on the scientific studies cited above, we propose pedagogical recommendations for preelementary and early primary school teachers (Year 1 - CP). The idea is to have their students practice, more systematically, various activities centered on patterns (Figure 7).

1. Focus on the abstract aspect of patterns

It is important that the child is aware that the pattern does not depend on the presence of specific elements. In other words, the teacher must help the child abstract the set of rules that make up the pattern and dissociate them from their physical manifestation. To promote this abstraction process, it may be useful to refer to the elements that make up the model using labels that have no connection to the particular object (for example, letters of the alphabet, ABAB, or invented names like "pouf bim pouf bim"). Research also suggests that it is important to emphasize the relationships between the elements that make up the pattern, focusing on aspects such as symmetry and order. Patterns are an excellent opportunity to introduce, as early as preschool, part of the vocabulary of numbers and geometry (pair, triplet; repetition, mirror, symmetrical; front, back; square, triangle...).

The links between mathematical, musical, reading, and writing skills likely depend, at least in part, on the shared use of abstract rules. Presenting students with examples of patterns from all these disciplines (geometric figures, numerical series, sound sequences, nursery rhymes, repetitions of graphic signs) could promote abstraction and enable students to generalize skills acquired in one modality to another. Although studies aimed at determining the concrete possibilities of generalization are still ongoing, it seems reasonable to stimulate the recognition and manipulation of patterns across all

2. Propose patterns in different ser

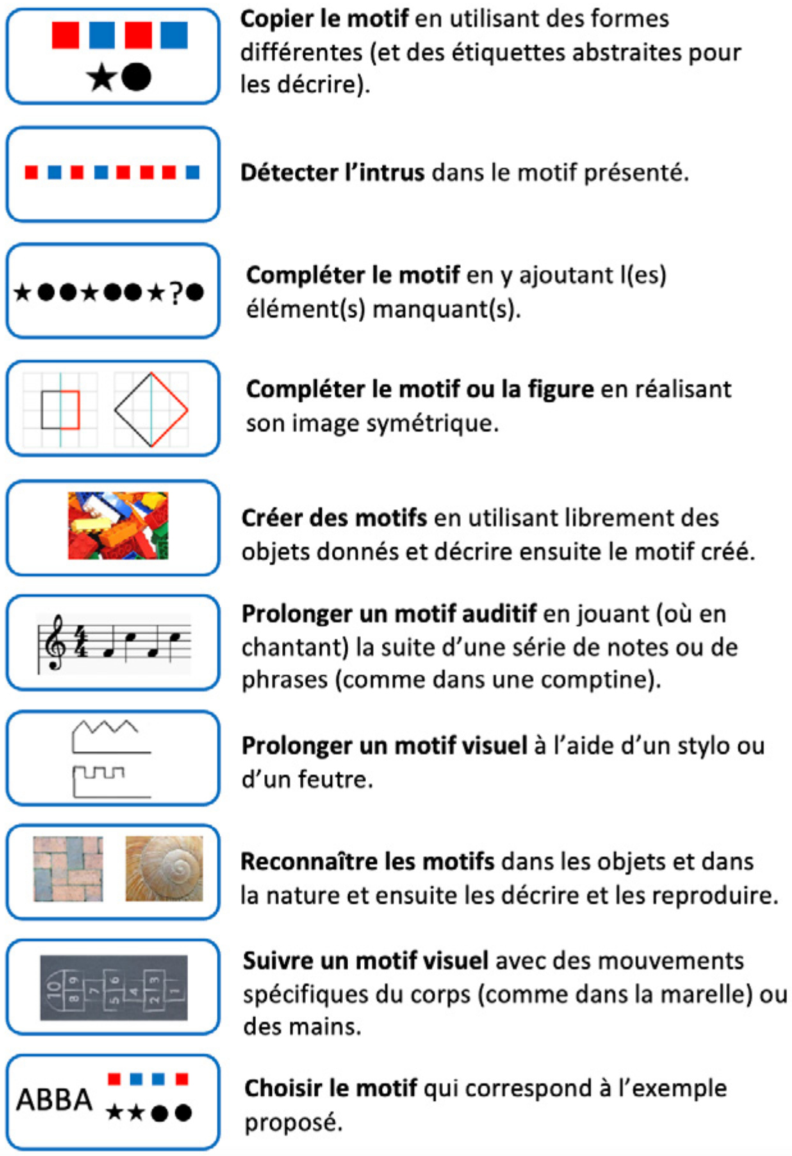

Figure 7. Examples of comprehension, manipulation and production activities with patterns.

disciplines and modalities that offer this opportunity.

The very idea that a sequence of sounds can be represented by a series of letters, from left to right, is the foundation of writing and musical notation. Playing games that involve writing down the patterns one hears or transcribing them using stickers or colored bricks can prepare students for reading as early as preschool.

3. Use different media, different patterns, and different tasks

Beyond the specific discipline or modality, it may be important to vary as much as possible the material used during activities involving the use of patterns. Let's take the most classic example, namely the presentation of patterns in a visual way: most of the studies presented in this text use small objects (beads) or colored bricks organized in a sequence. However, the exclusive use of these tools may not allow children to express all their understanding of patterns. Working memory has its limits, especially in the youngest children: for example, it is very difficult for a 4-year-old child to reproduce a pattern whose minimal unit consists of more than 3 or 4 different elements, simply because they would have difficulty remembering such a quantity of items. This does not mean, however, that they are not capable of recognizing much more complex patterns than the simple repetition and alternation programs presented in this text.

In this regard, activities using paper and pencil, where students are asked to continue a geometric border by following the pattern, seem particularly useful. In this case, the emphasis should also be placed not (or not only) on the strict adherence to motor gestures and precise relationships between graphic signs (in terms of, for example, size and dimension), but above all on the underlying abstract rule. This approach aims to highlight the recognition of structures and sequences beyond the mere improvement of fine motor skills.

4. Stimulate multiple proto-mathematical intuitions

Some preelementary classes limit themselves to very restricted and relatively simple patterns (a red bead, a yellow one, a red one, a yellow one…), and this without explicit reference to mathematical vocabulary. We propose, on the contrary, to use more complex patterns that stimulate students' imagination and abstraction, and to introduce them, more or less implicitly, to many proto-mathematical concepts. More concretely, we propose the following sequential and geometric patterns.

Sequential patterns

The proposed patterns should go beyond simple alternating series (of the ABAB type, i.e., with a single element per group) to include repetitions of groups of two (AABBAABB) or three elements (AAABBBAAABBB). Additionally, especially for older students, it may be useful to introduce series where elements alternate following an increasing or decreasing progression in the number of repetitions (such as ABAABBAAABBB or AAABBBAABBAB). Such series provide an opportunity to introduce, very early on, concepts that are notoriously difficult for many children to grasp, such as proportions and fractions. This can be done through questions to the child: "What do you think: did you use as many yellow beads as red ones? Why?" These sequences, presented for example as necklaces (Figure 8), can also be an opportunity to introduce mathematical concepts without explicit reference to numerical units and without requiring arithmetic calculations. Developing the pattern "ABBABB" encourages reflection on the notion of fractions ("Two out of three beads are red," "There are twice as many red beads as yellow ones," etc.).

The necklace on the left in Figure 8 should encourage the child to discover on their own that "half of the beads are blue." On the right, the repetition present in the pattern ABCABCABC can encourage them to discover certain decompositions of small whole numbers: "There are 3 groups of 3 beads, making 9 beads in total."

Geometric patterns

Geometric patterns are another excellent example of proto-mathematical activities. In addition to the activities already mentioned above (such as completing spirals or Greek borders), it may be useful to offer students activities based on assembling simple geometric shapes, a game that is becoming increasingly popular among educational and recreational activities available commercially (Figure 9). Assembling a hexagon, as shown in the figure, from different shapes can stimulate an intuitive

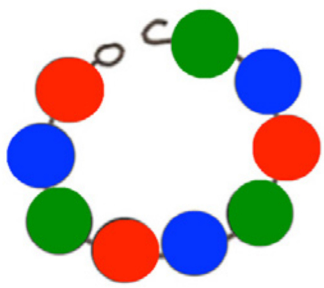

Figure 8. Examples of necklaces where beads follow sequential alternation rules.

understanding of various proto-mathematical concepts, including recursion ("12 repeated pink triangles form a hexagon"), fractions ("6 blue blocks occupy the same space as 12 pink blocks"), or symmetry ("In how many ways can I divide each hexagon to obtain two symmetrical images?").

Figure 9. Exemple of an assembly game of geometrical forms (here, 21st century pattern blocks)

Activities of this type also offer a significant collateral advantage: they are perceived as artistic and playful activities, which can reduce math-related anxiety and, perhaps, the gender bias. Research has shown the significant impact of the stereotype that mathematics is an activity reserved for boys. This stereotype deeply influences girls' math performance in school starting from first grade (*we invite you to read CSEN Note 3 on this topic*). However, when the same activity of copying a geometric figure is presented as "drawing" rather than "math," this stereotype disappears, and girls and boys achieve comparable performance[45]. Patterns thus seem to be an ideal domain for fostering a love of mathematics in both girls and boys.

5. In primary school, stimulate algebraic thinking with numerical patterns

In primary school, with the formal learning of mathematics and, more specifically, the introduction of arithmetic, more complex numerical patterns can be systematically incorporated into classroom activities. Indeed, most of the activities proposed so far in this note can be reimagined using numbers instead of sounds or colored blocks. For example, it could be useful to present students with simple number sequences such as 4, 7, 10, 13, 16, 19 and ask them to complete them. The challenge—and thus the interest—of such a sequence, where each successive element adds 3 units to the previous one, lies in actively stimulating the search for regularity within the sequence. Unlike simpler sequences like 2, 4, 6, 8, 10, where one might rely on memorized multiplication tables, this task requires identifying the underlying rule.

Other examples of such number sequences include: 19, 17, 15, 13... (subtracting 2 units), 2, 4, 8, 16, 32... (multiplying by 2), or even mixed sequences, where different rules alternate: 3, 6, 4, 7, 5, 8... (adding 3 units, then subtracting 2).

Searching for the rule in such sequences serves as an excellent intuitive introduction to algebraic thinking, which is typically formally introduced much later, in middle school. This approach leads to all sorts of interesting questions—for example, by continuing the sequence 4, 7, 10, 13, 16, 19..., can we reach the number 20? 100? 1,000? 10,000? Why or why not? Can we provide a general formula for all the numbers in the sequence? While these questions clearly go beyond the scope of first grade, they illustrate how familiarity with patterns, starting in preschool, can serve as an intuitive and accessible introduction to more advanced mathematical concepts, stimulating students' imagination.

Experiments based on exercises of this type are currently underway in French schools. Readers are invited to consult the following excellent article for further exploration of the topic: https://afdm.apmep.fr/rubriques/opinions/des-patterns-dans-les-classes.

6. Value the multiplicity of correct answers: A single pattern can follow different rules and be formulated in different ways

An objection that could be raised against the excessive use of activities centered on patterns is that it gives students the impression that there is only one correct answer, i.e., that each pattern is determined by a precise and unique rule. If this seems true for the simplest patterns of repetition and alternation or for patterns of a geometric nature, it is not always the case for more complex sequences. Let's take, for example, an activity that would consist of completing the following pattern: AB?ABB. The missing element could be a B if we recognize the repetition of the ABB pattern, or an A if we recognize a pattern based on a "growth" rule of the type ABAABBAAABBB.... Both patterns also seem equally plausible, as their complexity is similar. Through such examples, the student can be led to discover that the induction of the solution is not a simple problem, and thus enrich their repertoire of mental programs.

The teacher's role then becomes particularly important: they must show students the variety of rules that can be used to complete certain patterns, stimulating their exploration and comparison. This exploration should always be, at least in part, guided by the teacher; otherwise, students risk relying solely on their own intuitions, which may be limited.

It is also worth noting that it is entirely possible to find **different formulations** for a single pattern. For example, the increasing pattern ABAABBAAABBB... can be described as "produce one A and one B, then two A's and two B's, three A's and three B's, and so on," or as "alternate A and B, and with each alternation, add one more A and one more B." For a young child, it is not immediately obvious that these formulations are equivalent.

Convincing themselves—and others—of this equivalence can serve as a first introduction to mathematical proof.

7. Combine patterns with explicit teaching of mathematical concepts

A danger would be to introduce mathematical concepts only through the discovery, by students, of certain regularities. We would then fall into a naive application of what is called "discovery learning," which has repeatedly demonstrated its ineffectiveness[46]. The scientific literature suggests that an **explicit** teaching of mathematical tools is indispensable[47-49], as most students cannot discover highly abstract concepts on their own—concepts that took mathematicians centuries to invent. The Scientific Council of National Education has repeatedly insisted on this point. (*We invite you to read Letter No. 2 of "Le Passeur" and to watch the replay of a conference dedicated to explicit teaching at the following link: www.reseau-canope.fr/lenseignement-explicite.html*.)

We propose patterns solely as a concrete, intuitive, engaging, and early introduction to the abstraction processes that will, much later, form the foundations of far more elaborate concepts, such as bijection, group theory, symmetry, computer programs, and more. In preschool, it is, of course, not a matter of using these complex terms or their corresponding written notation but simply of planting the first seeds of understanding their nature in students.

It thus clearly appears that the activities proposed here: (1) Do not replace **explicit teaching of mathematical concepts**, which remains essential; (2) primarily concern the **awakening** of abstraction in preschool and the very beginning of primary school; (3) can also, later on, help introduce more advanced concepts (numerical series, two-dimensional tilings, etc.), and this up to the highest levels of mathematics (as evidenced by the very recent discovery of a new type of non-periodic tiling[50])—but always based on explicit teaching of the corresponding mathematical vocabulary and concepts.

Cognitive psychology and neuroscience of mental programs

Music, geometry, and mathematics are based on the use of patterns and structures combined according to rules that, to be understood, do not necessarily need to be translated into words13. Indeed, scientific research suggests that the practice of these disciplines (and particularly mathematics) relies on distinct brain regions separate from the areas involved in spoken or written language51,52. Many studies support the hypothesis that our brain uses a "language of thought" that does not require words53 but operates based on "mental programs," somewhat like computer languages, which it derives through induction from the external world54.

In this hypothesis, the pattern 11223344 would be represented in memory by a kind of mental program: "for i = 1:4, print i twice." When faced with a regular shape or pattern, the human brain cannot help but search for the appropriate program, meaning the operations needed to repeat or complete it. For example, when confronted with a square, our mind "induces" that, to reproduce it, it is necessary to create four sides of equal length forming four right angles.

Let's take another example: when we hear the alternation of notes in Figure 3, we recognize an increasing pattern even before we can formalize in words what we have heard. Recent studies12 also seem to suggest that humans, when asked to recognize or memorize such patterns, tend to find the simplest and most efficient rule that allows them to compress the information as much as possible. In other words, instead of memorizing the exact number of occurrences of each note in the pattern in Figure 3 (a do followed by a fa, followed by two dos, followed by two fas, and so on), we effortlessly recognize the following program: "alternate do and fa, and with each alternation, add one do and one fa."

The ability to use such mental programs is present in all humans, adults and children alike, regardless of their level of education4,13. This ability appears to be unique to the human brain: while animals can learn certain sequences and rules, they do not converge toward the simplest minimal solution4 that we immediately recognize as "a square," "an increasing alternation," or "a zigzag."

Key takeaways

- Patterns are sequences of items (objects, numbers, sounds…) in which we recognize an **order**, **regularity**, and therefore **predictability**.
- Many early childhood and primary school teachers propose exercises based on the recognition and use of patterns to improve **fine motor skills** and **writing**, without necessarily perceiving their mathematical dimension.
- Scientific research suggests (without definitively proving) that school activities with patterns can improve **mathematical skills**, as they constitute an excellent example of proto-mathematical abstraction.
- During their development, **children progressively identify increasingly complex regularities** that allow them to understand, extend, and transpose patterns.
- To emphasize the abstract nature of the rules underlying patterns, we suggest **presenting them in different ways:** the same ABABAB pattern can be presented to students in the form of sounds, beads, letters, numbers, etc.
- Beyond simple ABABAB alternations, preelementary school students should be confronted with more complex regularities, for example, repetitions (AABBAABB…), increasing series (ABAABBAAABBB…), or involving more than two elements (ABCDABCD…; AABBCC…).
- We advise teachers to propose **different tasks** to their students: copying a pattern, extending and prolonging it, finding the intruder in a sequence, completing a figure with its symmetrical image, completing a series of numbers…
- Games and exercises with **geometric figures** are an excellent way to implicitly introduce important mathematical concepts (symmetry, half, pair, set, fraction…).
- The **playful, creative, and artistic nature** of exercises with patterns could be favorable to mathematical development in both boys and girls, reducing the stereotypes associated with this discipline.
- We suggest that teachers value the **multiple abstract formulations** that students can propose for a pattern, but always guide them **explicitly** in understanding the underlying mathematical concepts, to which patterns are only a first intuitive approach.

Bibliography

1. Steen, L. A. The Science of Patterns. *Science* 240, 611–616 (1988).

2. Zazkis, R. & Liljedahl, P. Generalization of patterns: the tension between algebraic thinking and algebraic notation. *Educational Studies in Mathematics* 49, 379–402 (2002).

3. Smith, M. S., Hillen, A. F. & Catania, C. L. Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms. *MTMS* 13, 38–44 (2007).

4. Sablé-Meyer, M. et al. Sensitivity to geometric shape regularity in humans and baboons: A putative signature of human singularity. *Proc Natl Acad Sci USA* 118, e2023123118 (2021).

5. Starkey, G. S. & McCandliss, B. D. The emergence of "groupitizing" in children's numerical cognition. *Journal of Experimental Child Psychology* 126, 120–137 (2014).

6. Ciccione, L. & Dehaene, S. Grouping Mechanisms in Numerosity Perception. *Open Mind* 4, 102–118 (2020).

7. Ciccione, L. & Dehaene, S. Can humans perform mental regression on a graph? Accuracy and bias in the perception of scatterplots. *Cognitive Psychology* 128, 101406 (2021).

8. Ciccione, L., Sablé-Meyer, M. & Dehaene, S. Analyzing the misperception of exponential growth in graphs. *Cognition* 225, 105112 (2022).

9. Ciccione, L., Dehaene, G. & Dehaene, S. Outlier detection and rejection in scatterplots: Do outliers influence intuitive statistical judgments? *Journal of Experimental Psychology: Human Perception and Performance* 49, 129–144 (2023).

10. Ciccione, L. *et al.* Trend judgment as a perceptual building block of graphicacy and mathematics, across age, education, and culture. *Scientific Reports* (2023).

11. Schulz, E., Tenenbaum, J. B., Duvenaud, D., Speekenbrink, M. & Gershman, S. J. Compositional inductive biases in function learning. *Cognitive Psychology* 99, 44–79 (2017).

12. Sablé-Meyer, M., Ellis, K., Tenenbaum, J. & Dehaene, S. A language of thought for the mental representation of geometric shapes. *Cognitive Psychology* 139, (2022).

13. Dehaene, S., Al Roumi, F., Lakretz, Y., Planton, S. & Sablé-Meyer, M. Symbols and mental programs: a hypothesis about human singularity. *Trends in Cognitive Sciences* 26, 751–766 (2022).

14. Wijns, N., Torbeyns, J., De Smedt, B. & Verschaffel, L. Young Children's Patterning Competencies and Mathematical Development: A Review. in *Mathematical Learning and Cognition in Early Childhood* (eds. Robinson, K. M., Osana, H. P. & Kotsopoulos, D.) 139–161 (Springer International Publishing, 2019). doi:10.1007/978-3-030-12895-1_9.

15. Martin, L. *et al.* Abstract representations of small sets in newborns. *Cognition* 226, 105184 (2022).

16. Verdine, B. N. *et al.* Deconstructing Building Blocks: Preschoolers' Spatial Assembly Performance Relates to Early Mathematical Skills. *Child Dev* 85, 1062–1076 (2014).

17. Rittle-Johnson, B., Fyfe, E. R., McLean, L. E. & McEldoon, K. L. Emerging Understanding of Patterning in 4-Year-Olds. *Journal of Cognition and Development* 14, 376–396 (2013).

18. Fyfe, E. R., McNeil, N. M. & Rittle-Johnson, B. Easy as ABCABC: Abstract Language Facilitates Performance on a Concrete Patterning Task. *Child Dev* 86, 927–935 (2015).

19. Kotovsky, L. & Gentner, D. Comparison and Categorization in the Development of Relational Similarity. *Child Development* 67, 2797 (1996).

20. Papic, M. M., Mulligan, J. T. & Mitchelmore, M. C. Assessing the Development of Preschoolers' Mathematical Patterning. *JRME* 42, 237–268 (2011).

21. Gadzichowski, K. M. Patterning Abilities of First Grade Children: Effects of Dimension and Type. *CE* 03, 632–635 (2012).

22. Jirout, J. J. & Newcombe, N. S. Building Blocks for Developing Spatial Skills: Evidence From a Large, Representative U.S. Sample. *Psychol Sci* 26, 302–310 (2015).

23. Vanluydt, E., Wijns, N., Torbeyns, J. & Van Dooren, W. Early childhood mathematical development: the association between patterning and proportional reasoning. *Educ Stud Math* 107, 93–110 (2021).

24. Lee, K., Ng, S. F., Bull, R., Pe, M. L. & Ho, R. H. M. Are patterns important? An investigation of the relationships between proficiencies in patterns, computation, executive functioning, and algebraic word problems. *Journal of Educational Psychology* 103, 269–281 (2011).

25. Collins, M. A. & Laski, E. V. Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly* 32, 204–214 (2015).

26. Zippert, E. L., Clayback, K. & Rittle-Johnson, B. Not Just IQ: Patterning Predicts Preschoolers' Math Knowledge Beyond Fluid Reasoning. *Journal of Cognition and Development* 20, 752–771 (2019).

27. Kidd, J. K. *et al.* Effects of Patterning Instruction on the Academic Achievement of 1st-Grade Children. *Journal of Research in Childhood Education* 27, 224–238 (2013).

28. Pasnak, R. Empirical Studies of Patterning. *PSYCH* 08, 2276–2293 (2017).

29. Warren, E. & Cooper, T. Repeating Patterns and Multiplicative Thinking: Analysis of Classroom Interactions with 9 -Year- Old Students that Support the Transition from the Known to the Novel. *The Journal of Classroom Interaction* 41, (2007).

30. Wijns, N., Verschaffel, L., De Smedt, B., De Keyser, L. & Torbeyns, J. Stimulating preschoolers' focus on structure in repeating and growing patterns. *Learning and Instruction* 74, 101444 (2021).

31. Burgoyne, K., Witteveen, K., Tolan, A., Malone, S. & Hulme, C. Pattern Understanding: Relationships With Arithmetic and Reading Development. *Child Dev Perspect* 11, 239–244 (2017).

32. Planton, S. *et al.* A theory of memory for binary sequences: Evidence for a mental compression algorithm in humans. *PLoS Comput Biol* 17, e1008598 (2021).

33. Zentner, M. & Eerola, T. Rhythmic engagement with music in infancy. *Proc. Natl. Acad. Sci. U.S.A.* 107, 5768–5773 (2010).

34. Geist, K., Geist, E. A. & Kuznik, K. Young Children Learning Mathematics through Beat, Rhythm, and Melody. *Young Children* (2012).

35. Edelson, R. J. & Johnson, G. Music Makes Math Meaningful. *Childhood Education* 80, 65–70 (2003).

36. Klemish, J. J. A Comparative Study of Two Methods of Teaching Music Reading to First-Grade Children. *Journal of Research in Music Education* 18, 355–364 (1970).

37. Planton, S. & Dehaene, S. Cerebral representation of sequence patterns across multiple presentation formats. *Cortex* 145, 13–36 (2021).

38. Graziano, A. B., Peterson, M. & Shaw, G. L. Enhanced learning of proportional math through music training and spatial-temporal training. *Neurological Research* 21, 139–152 (1999).

39. Spelke, E. Effects of music instruction on developing cognitive systems at the foundations of mathematics and science. *Learning, arts, and the brain* 17, (2008).

40. Holmes, S. & Hallam, S. The impact of participation in music on learning mathematics. *London Review of Education* (2017) doi:10.18546/LRE.15.3.07.

41. Mehr, S. A., Schachner, A., Katz, R. C. & Spelke, E. S. Two Randomized Trials Provide No Consistent Evidence for Nonmusical Cognitive Benefits of Brief Preschool Music Enrichment. *PLoS ONE* 8, e82007 (2013).

42. Gordon, R. L., Fehd, H. M. & McCandliss, B. D. Does Music Training Enhance Literacy Skills? A Meta-Analysis. *Front. Psychol.* 6, (2015).

43. Montgomery, D. Teaching Prereading Skills through Training in Pattern Recognition. *The Reading Teacher* 30, (1977).

44. Swaminathan, S. & Schellenberg, E. G. Musical ability, music training, and language ability in childhood. *Journal of Experimental Psychology: Learning, Memory, and Cognition* 46, 2340–2348 (2020).

45. Huguet, P., Brunot, S. & Monteil, J. M. Geometry versus drawing: Changing the meaning of the task as a means to change performance. *Social Psychology of Education* 4, 219–234 (2001).

46. Mayer, R. E. Should There Be a Three-Strikes Rule Against Pure Discovery Learning? *American Psychologist* 59, 14–19 (2004).

47. Guilmois, C. Efficacité de l'enseignement socioconstructiviste et de l'enseignement explicite en éducation prioritaire : Quelle alternative pour apprendre les mathématiques ? (Antilles, 2019).

48. Guilmois, C., Popa-Roch, M., Clément, C., Bissonnette, S. & Troadec, B. Effective numeracy educational interventions for students from disadvantaged social background: a comparison of two teaching methods. *Educational Research and Evaluation* 1–21 (2020).

49. Archer, A. L. & Hughes, C. A. *Explicit Instruction: Effective and Efficient Teaching*. (The Guilford Press, 2011).

50. Smith, D., Myers, J. S., Kaplan, C. S. & Goodman-Strauss, C. An aperiodic monotile. Preprint at http://arxiv.org/abs/2303.10798 (2023).

51. Amalric, M. & Dehaene, S. Cortical circuits for mathematical knowledge: evidence for a major subdivision within the brain's semantic networks. *Phil. Trans. R. Soc. B* 373, 20160515 (2018).

52. Amalric, M. & Dehaene, S. Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences* 113, 4909–4917 (2016).

53. Fedorenko, E. & Varley, R. Language and thought are not the same thing: evidence from neuroimaging and neurological patients: Language versus thought. *Ann. N.Y. Acad. Sci.* 1369, 132–153 (2016).

54. Lake, B. M., Salakhutdinov, R. & Tenenbaum, J. B. Human-level concept learning through probabilistic program induction. *Science* 350, 1332–1338 (2015).

Find all CSEN's publications on:

csen.education.gouv.fr